Bouncing universe with modified dispersion relation

Wen-Jian Pan [1][*] and Yong-Chang Huang [1][†]

[1] *Institute of theoretical physics,Beijing University of Technology,Beijing,100124,China*

Abstract

In this paper, employing the modified dispersion relation, we have derived the general modified Friedmann equations and the corresponding modified entropy relations for the Friedmann-Robertson-Walker (FRW) Universe. In this setup, we find that when the big bounce happens, its energy scale and its corresponding modified entropy behavior are sensitive to the value of k. In contrast to the previous work with $k = 0$, our work mainly demonstrates that the bouncing behavior for the closed Universe with $k = 1$ appears at the normal energy limit of the modified dispersion relation introduced, and when bouncing phenomenon is in presence, its modified entropy is just equal to zero. Surprisingly, when $k = -1$, the bouncing behavior is in absence.

I. INTRODUCTION

General relativity plays an important role in deeply linking the geometric structure of spacetime with the distribution of matter sources. This remarkable theory provides a powerful perspective to understand the nature of gravity and describe the evolution of the Universe. With the cosmological principle, the standard cosmological model can be derived successfully from general relativity. However, the cosmological singularity problem cannot be avoided effectively in this model. To solve this singularity problem, theorists widely turn to appealing for the quantum gravity. Unfortunately, up to now, one has not yet established a complete and consistent quantum theory of gravity, which results in that the singularity problem cannot be solved at first principle level. As a result, people need to seek for some important quantum properties in string theory and loop quantum gravity as important mechanisms to relieve or solve this problem at a phenomenological level.

Nowadays people generally believe that the combination of general relativity and quantum mechanics will provide a fundamental minimal length or the maximal energy, which is referred as a feature of quantum gravity. The developments of string theory and loop quantum gravity have greatly intensified this belief. In particular, based on Loop Quantum Gravity there have been great interests in some modification of dispersion relations [1–3], and Generalized Uncertainty Principles(GUP)[4, 5] have been taken into account in the literature on string theory [6, 7] and on noncommutative geometry[8]. The forms of the modified dispersion relation (MDR) and ones of GUP can be viewed as the alternative mechanisms to handle the black hole problems, such as the singularity, the "thermodynamics", the evaporation and the "information paradox" and so on. In the recent years, the implication and application of the effects of quantum gravity have attracted a great deal of attentions [9–32] (for recent reviews we refer to [33–35]).

In the semi-classical limit an intuitive scenario was proposed to avoid the cosmological singularity problem in standard cosmology by a big bounce [36, 37] that was implemented by modifying the standard Friedmann equation. Recently, this scenario has been extensively investigated in various approaches [38–48]. In particular, based on the assumption that the gravitational theory can be described by entropy force [49], which has been widely interested by many theorists [40, 50–63], the authors in [40] proposed that an interesting setup was that the modified dispersion relation prevented the energy density of the matter contents from diverging at high energy level such that the modified Friedmann equation with the

bouncing effect can be derived successfully from the Clausius relation, where the singularity of the corresponding spacetime was free.

In the previous work [40], the authors had discussed the relevant bouncing behavior and the corresponding entropy behavior for spatial flat universe by the modified dispersion relation. Now we wonder whether similar interesting behaviors for the cases with non-vanishing k will happen. We expect to provide an answer to this problem in a parallel way. It turns out that using the modified dispersion relation in [40] to prevent the energy density of the matter from diverging, we can obtain the general modified Friedmann equations and the corresponding modified entropy relations for the FRW Universe. When $k = 0$, our results naturally reduce to that considered in [40]. In particular, we find that the modified Friedmann equation contains bouncing behavior and the corresponding entropy is non-negative for the closed universe. More precisely, its bounce appears at the normal energy bound of the MDR, namely $E = \frac{\pi}{2\eta l_p}$. This implies that when bouncing phenomenon is in presence, its entropy is just equal to zero, which is quite different from the result in the previous work [40]. In addition, we have noted that when $k = -1$, the bouncing behavior is in absence. Our results have strongly suggested that k plays an important role on the bouncing behavior of the Universe.

Our paper is organized as follows: in Sect. II, we briefly review on the description of entropy force for the Friedmann-Robertson-Walker universe; in Sect. III, our task is to determine the modified universe equations with the MDR in detail and discuss the corresponding entropy relations; in Sect. IV, we discuss the bouncing effects of the universe with various values of k in detail; Sect. V gives conclusions and discussions. In Appendix, we present a detailed calculation to give out the reason why the bouncing behavior for the closed universe occurs at $a_0 = \eta l_p$. Here we have set $\hbar = c = 1$ for convenience.

II. THE ENTROPY FORCE DESCRIPTION

In the section, we will briefly recall some ingredients for deriving the Friedmann-Robertson-Walker universe from the relation of the entropy force. Let us begin with introducing the Friedmann-Robertson-Walker metric which is

$$ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2], \tag{1}$$

where k values can be taken $-1, 0$ and 1 which correspond to the open, flat and closed Universe, respectively, and $a(t)$ is the scale factor of the Universe. This spacetime geometry implies that the apparent horizon [62] can be defined as

$$r_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}. \tag{2}$$

In analogy with black hole thermodynamics, for the apparent horizon we can correspondingly define its entropy as

$$S = \frac{A}{4G} = \frac{\pi}{G(H^2 + \frac{k}{a^2})}, \tag{3}$$

as well as the temperature

$$T = \frac{1}{2\pi r_A}, \tag{4}$$

where $A = 4\pi r_A^2$ is the apparent horizon area. In the viewpoint of entropy force, utilizing the Clausius relation, $\delta Q = T\delta S$, and integrating the both sides of this relation, one can naturally obtain Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho_t, \tag{5}$$

where ρ_t is total energy density. Here we have successfully derived Friedmann equation from the Clausius relation via assuming that our universe is homogeneous and isotropic, and treating one as a thermodynamical system in equilibrium. It is obvious that the singularity still exists. Thus, in the next sections, after taking a specific MDR into account, we expect to apply the corresponding entropy force prescription to construct the modified universe equations with any spatial curvature, which may include the bouncing universe behavior, so that the singular problem can be free.

III.   MODIFIED UNIVERSE EQUATIONS WITH MODIFIED DISPERSION RELATION

In this section, we mainly concentrate on the modified dispersion relation how it causes the modification of the FRW Universe equations with a spatial curvature k in semi-classical limit. Here we will introduce the quantum gravitational effect at the phenomenological level. Let us start with the modified dispersion relation. Based on Doubly Special Relativity, a general modified dispersion relation proposed in [11, 12] has a following form,

$$E^2 f^2(l_p E) - p^2 g^2(l_p E) = m^2, \tag{6}$$

where E, p and m are the energy, momentum and rest mass of the particle, respectively, $l_p^2 = 8\pi G$. The above MDR has been widely used to study black hole physics and rainbow universe [15–19, 38, 39]. At low energy limit, namely $l_p E \ll 1$, one naturally requires that the two general functions $f(l_p E), g(l_p E) \sim 1$, so that the above dispersion relation reduces to the standard Einstein energy-momentum relation. The authors of the previous work [40], in fact, have chosen the two specific functions for the modified dispersion relation

$$f^2(l_p E) = \frac{\sin^2(\eta l_p E)}{(\eta l_p E)^2}, \qquad g^2(l_p E) = 1, \tag{7}$$

then Eq.(6) gives rise to the following form,

$$\frac{1}{\eta l_p} \sin(\eta l_p E) = \sqrt{(p^2 + m^2)}, \tag{8}$$

where η is dimensionless parameter which is bigger than zero. They have omitted the negative energy branch. The right side of the equation is not less than zero, so that the value of energy is in the range $[0, \frac{\pi}{\eta l_p}]$. Obviously, this MDR can be separated into two branches, namely $[0, \frac{\pi}{2\eta l_p}]$ and $[\frac{\pi}{2\eta l_p}, \frac{\pi}{\eta l_p}]$, respectively. It is easy to find that in the different branches, the monotonous behavior of their momentum is remarkably distinguishing, as the energy is increasing. More explicitly, for the particle energy in branch $[0, \frac{\pi}{2\eta l_p}]$, its momentum p is increasing as the energy is growing, while for the particle energy in branch $[\frac{\pi}{2\eta l_p}, \frac{\pi}{\eta l_p}]$, p has a strangely and monotonously decaying behavior. Thus we call the latter branch as the anomalous energy region. In particular, for the rest mass $m = 0$, in between the two branches, when particle energy passes through $E = \frac{\pi}{2\eta l_p}$, its momentum reaches a peak value, namely $\frac{1}{\eta l_p}$; whereas when $E = \frac{\pi}{\eta l_p}$, its momentum p vanishes. In order to introduce the bouncing effect in the Universe, we need to establish further the variation relation of the above equation (8) that has a following form

$$\delta E = \pm \frac{1}{\sqrt{1 - \sin^2(\eta l_p E)}} \frac{p}{\sqrt{(p^2 + m^2)}} \delta p, \tag{9}$$

where the plus sign " $+$ " and the minus sign " $-$ " stand for choosing the energy value in the range $[0, \frac{\pi}{2\eta l_p}]$, and $[\frac{\pi}{2\eta l_p}, \frac{\pi}{\eta l_p}]$, respectively. For the high energy limit or massless particle, the above equation can be expressed as

$$\delta E = \pm \frac{1}{\sqrt{1 - (\eta l_p p)^2}} \delta p. \tag{10}$$

Later, we will see that the above relation plays an important role in deriving the modified universe equation which may relieve the singularity problem. When the modified dispersion

relation is taken into account, the modified Friedmann equation can be derived via an integration relation in [62]

$$\frac{8\pi G}{3}\rho = -\frac{\pi}{G}\int S'(A)(\frac{4G}{A})^2 dA, \tag{11}$$

where $S'(A) = \frac{dS}{dA}$ is related to the modified dispersion relation, while for ordinary case it is a well-known constant, namely $\frac{1}{4G}$. Now let us consider how such modified ratio of the entropy to the area on the apparent cosmological horizon is derived out, and then how it governs the modified Friedmann equation such that the singularity of the universe can be free. We treat our universe as a thermodynamical system in equilibrium and suppose a quantum process in which a single massless particle with energy E escapes through its apparent horizon. This causes correspondingly the minimal entropy change [14, 17, 19], which should satisfy Clausius relation,

$$\delta S_{min} = dS = \frac{\delta Q}{T} = \pm\frac{2\pi}{\sqrt{1 - \frac{4\pi(\eta l_p)^2}{A}}}, \tag{12}$$

here we have used the identification relations $\delta Q \sim E \sim \delta E$ as well as $p \sim \delta p$, and the Heisenberg uncertainty relation $\delta p \sim \frac{1}{\delta x} \sim \frac{1}{r_A}$ [20]. Moreover, we have also admitted a physical fact that for the particle in the quantum process there is an intrinsic uncertainty position as its Compton wavelength that is identified with the apparent horizon of the universe. In this quantum process, correspondingly, one assumes that causing the minimal area change of the apparent horizon is $\delta A_{min} = l_p^2 = 8\pi G$ [40](and references therein). Identifying dA with δA_{min} and using Equation (12), we have

$$S'(A) = \frac{dS}{dA} = \frac{\delta S_{min}}{\delta A_{min}} = \pm\frac{1}{4G}\frac{1}{\sqrt{1 - \frac{4\pi(\eta l_p)^2}{A}}}. \tag{13}$$

Let us first consider the plus case. Putting this equation into the integration relation (11), we can obtain the modified Friedmann equation,

$$\frac{8\pi G}{3}\rho = -\frac{2}{\eta^2 l_p^2}\sqrt{1 - \frac{4\pi\eta^2 l_p^2}{A}} + C_1, \tag{14}$$

when $4\pi\eta^2 l_p^2 \ll A$, Eq.(14) should reduce to the standard Universe equation, which is $\frac{8\pi G}{3}\rho + \frac{\Lambda}{3} = H^2 + \frac{k}{a^2}$, thus the integral constant can be figured out

$$C_1 = \frac{2}{\eta^2 l_p^2} - \frac{\Lambda}{3}. \tag{15}$$

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So the modified kinematic Friedmann equation can be determined as

$$\frac{8\pi G}{3}\rho_t = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} = \frac{2}{\eta^2 l_p^2}(1 - \sqrt{1 - \frac{4\pi\eta^2 l_p^2}{A}}). \tag{16}$$

Making both sides of Eq.(16) square and simplifying it, we can obtain an alternative form as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho_t(1 - \frac{\rho_t}{\rho_c}), \tag{17}$$

where $\rho_c = \frac{12}{\eta^2 l_p^4}$. With the use of the continuity equation $\dot{\rho}_t + 3H(\rho_t + P_t) = 0$, the modified dynamical Friedmann equation can be calculated out

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\rho_t(1 - \frac{\rho_t}{\rho_c}) - 4\pi G(\rho_t + P_t)(1 - 2\frac{\rho_t}{\rho_c}), \tag{18}$$

where P_t is the total pressure.

The modified entropy-area relation for energy range $[0, \frac{\pi}{2\eta l_p}]$ can be expressed as

$$S_M = \frac{A}{4G}\sqrt{1 - \frac{4\pi\eta^2 l_p^2}{A}} + \frac{\pi\eta^2 l_p^2}{G}\ln[\sqrt{\frac{A}{4\pi\eta^2 l_p^2}} + \sqrt{\frac{A}{4\pi\eta^2 l_p^2} - 1}] + D_1, \tag{19}$$

when $\eta \to 0$, the modified entropy-area relation reduces to the normal result, namely $\frac{A}{4G}$, which can give rise to the integral constant $D_1 = 0$.

Similarly, for the energy range $[\frac{\pi}{2\eta l_p}, \frac{\pi}{\eta l_p}]$ we can work out the modified Universe equation

$$\frac{8\pi G}{3}\rho = \frac{2}{\eta^2 l_p^2}\sqrt{1 - \frac{4\pi\eta^2 l_p^2}{A}} + C_2. \tag{20}$$

Here we require that when $A = 4\pi\eta^2 l_p^2$ corresponding to the energy value $\frac{\pi}{2\eta l_p}$, Eq.(20) can match smoothly with Eq.(14) so that C_2 should be equal to C_1, namely $C_2 = C_1 = \frac{2}{\eta^2 l_p^2} - \frac{\Lambda}{3}$. In the parallel way, with the use of the above equation and continuity equation, it is not hard to reproduce the modified universe equations which are the same as Eqs.(17) and (18), respectively. Similarly, the corrected entropy relation also becomes

$$S_M = -\frac{A}{4G}\sqrt{1 - \frac{4\pi\eta^2 l_p^2}{A}} - \frac{\pi\eta^2 l_p^2}{G}\ln[\sqrt{\frac{A}{4\pi\eta^2 l_p^2}} + \sqrt{\frac{A}{4\pi\eta^2 l_p^2} - 1}] + D_2. \tag{21}$$

Here we also require that the modified entropy-area relations in both Eqs.(19) and (21) are the same, when $A = 4\pi\eta^2 l_p^2$. Thus we can obtain the integral constants satisfy a relation $D_2 = D_1 = 0$. It is easy to check that when $A = 4\pi\eta^2 l_p^2$ the value of entropy-area vanishes, namely $S_M = 0$.

IV. THE BOUNCING BEHAVIORS IN THE MODIFIED UNIVERSE EQUATIONS

In this section, we mainly explore the bouncing behavior for the modified Friedmann's equations. In the bouncing universe scenarios, the most appealing feature is no initial singularity of the universe. In these bouncing paradigms, ones usually argue that there should be a nonsingular connection between the two distinguishing phases that are the contraction phase and the expansion phase in order. More explicitly, the evolution of the cosmological scale factor from the contraction phase with $\dot{a} < 0$ to the expansion phase with $\dot{a} > 0$, goes through its minimal value with non-vanishing value referred as a critical point. In such process, some effects (quantum effects) that become the dominant ingredients would prevent the universe from collapsing into a singularity and then drive our universe to accelerate expansion. As a result, at the critical point the cosmological scale reaches the minimal nonzero value($a_0 > 0$) which has $H_0 = 0$ and $\ddot{a}_0 > 0$[40]. Thus, we can reasonably view them as the bouncing conditions. After taking the first bouncing condition into account, namely $H_0 = 0$, solving reversely modified Friedmann equation (17) gives rise to

$$\rho_{t\pm} = \frac{1}{2}\rho_c(1 \pm \sqrt{1 - \frac{k\eta^2 l_p^2}{a_0^2}}), \tag{22}$$

which should meet the constraints $\rho_{t\pm} > 0$ and $r_A \geq 0$. Here we have denoted the corresponding scale factor as a_0 when the Hubble parameter vanishes. Making use of Eqs.(17) and (22), the modified dynamical Friedmann equation is correspondingly represented by

$$\frac{\ddot{a}_0}{a_0} = \frac{k}{a_0^2} \pm 4\pi G \sqrt{1 - \frac{k\eta^2 l_p^2}{a_0^2}}(\rho_{t\pm} + P_t), \tag{23}$$

where the signs " $\pm$ " in the above equation correspond to $\rho_{t\pm}$, respectively. There are two points what we would like to emphasize. Firstly, ρ_{t-} is an ordinary energy density in the energy scale region $[0, \frac{\pi}{2\eta l_p}]$, while ρ_{t+} is anomalous energy density in $[\frac{\pi}{2\eta l_p}, \frac{\pi}{\eta l_p}]$. When taking energy density relation in (22) to calculate the cosmic acceleration in between the contracting phase and the expansion phase, we have to use the corresponding relation in (23). Secondly, when $k \neq 0$, this means there is an undetermined degree of the freedom, namely the scale factor a_0. Now we are going to divide them into three cases, namely $k = -1$, $k = 0$ and $k = 1$, respectively, and check whether they really satisfy the constraints and the other bouncing condition or not.

- For $k = -1$, when requiring this case to satisfy the first bouncing condition, one easily finds that r_A becomes an imaginary number, which is not consistent with the constraint $r_A \geq 0$. Thus for this case the bouncing solution does not exist!

- For $k = 0$, we have

$$\rho_{t+} = \rho_c; \rho_{t-} = 0. \tag{24}$$

When our Universe reaches high energy bouncing critical point which corresponds to the total energy density is rather $\rho_{t+} = \rho_c$ than $\rho_{t-} = 0$, this gives out the dynamical Universe equation which is

$$\frac{\ddot{a}_0}{a_0} = 4\pi G(\rho_c + P_t) > 0, \tag{25}$$

it is just the result of the previous work[40].

- For $k = 1$ the total energy density equation (22) naturally reduces to the following form

$$\rho_{t\pm} = \frac{\rho_c}{2}(1 \pm \sqrt{1 - \frac{\eta^2 l_p^2}{a_0^2}}). \tag{26}$$

It seems that there are two bouncing solutions. Based on the definition of the apparent horizon and the MDR, we can find the scale factor is no less than ηl_p. In Appendix, according to the bounce conditions, we can show that the bouncing effect of the Universe is presented at the minimum scale value that means the momentum reaches the maximum value corresponding to the critical point $E = \frac{\pi}{2\eta l_p}$ such that in fact the above equation (26) can only take the same value, namely $\rho_{t-} = \rho_{t+} = \frac{\rho_c}{2}$ with $a_0^2 = \eta^2 l_p^2$. After the big bounce of our Universe happening, the total density will decrease monotonously with our Universe expanding. When the total energy density climbs up the value $\rho_{t-} = \rho_{t+} = \frac{\rho_c}{2}$, Eq.(23) can be rewritten as

$$\frac{\ddot{a}_0}{a_0} = \frac{1}{\eta^2 l_p^2} > 0. \tag{27}$$

Thus for this case the bounce of the Universe can exist in the normal energy region $[0, \frac{\pi}{2\eta l_p}]$ without anomalous negative entropy!

Here it is worth noting that for the case $k = 0$ the bouncing solution occurs at the critical point with maximum energy value $\frac{\pi}{\eta l_p}$, but with negative entropy in [40]. For $k = 1$ the bouncing phenomenon can be presented at the energy value $\frac{\pi}{2\eta l_p}$ corresponding to the

maximum momentum $\frac{1}{\eta l_p}$. This means that the modified entropy-area relation is rather Eq.(19) than Eq.(21). More precisely, when the universe performs its bouncing behavior in high energy limit, its corresponding entropy-area relation is just equal to zero. After the big bounce, its modified entropy will increase monotonously with our Universe expanding. For $k = -1$ the bouncing one is absent, since it can not meet the constraint condition $r_A \geq 0$.

V.   CONCLUSIONS AND DISCUSSIONS

In this paper, along with the spirit in [40], assuming that both the Clausius relation and the modified dispersion relation keep effective on the apparent horizon, we have derived the general modified universe equations and the corresponding modified entropy relations for the FRW Universe. From the modified universe equations, we find that the bouncing behavior is presence for the closed Universe, besides the spatial flat Universe, so that the cosmological singularity problem can be solved for these cases at the phenomenological level. In particular, in contrast to the bouncing solution of the spatial flat universe with $k = 0$ to occur at the maximum energy value $\frac{\pi}{\eta l_p}$ of the MDR (8), but with negative entropy in [40], we have shown that the bouncing behavior of the closed Universe with $k = 1$ explicitly appears at the energy value $\frac{\pi}{2\eta l_p}$, without negative entropy. More precisely, when the closed universe performs the bouncing behavior at the critical energy point $\frac{\pi}{2\eta l_p}$, the corresponding entropy-area relation is just equal to zero. Surprisingly the bouncing solution of $k = -1$ is absent, since it can not satisfy the constraint condition $r_A \geq 0$.

However, it is well-known that the entropy relation of the Universe is an open question in the bouncing or cyclic universe scenarios, if we respect the second law of the thermodynamics saying that the total entropy of the universe never decreases as time flows. When the closed Universe evolves from the contraction phase to the expansion phase, Eq.(19) suggests that the corresponding entropy-area relation has diminishing behavior and increasing behavior, respectively. These facts tell us that the entropy behavior in the contraction phase does not agree with the second law, but one in the expansion phase agrees with the second law. Thus, in analogy with the previous bouncing or cyclic universe scenarios, the well-known entropy problem that violates the second law is still in presence in our scenario. A deep understanding on how to link the diminishing behavior of the entropy in the contracting phase and the second law of the thermodynamics is still lacking. There might be an approach to solve this problem via considering the generalized second law of thermodynamics which has been

discussed in many universe models[64–67]. Under complying with the second law of the thermodynamics, it should be interesting to seek for some new modified dispersion relations and new physical mechanism to overcome or solve the singularity problem of the Universe. This aspect is left for future works.

Acknowledgments

Appendix

In this Appendix, we present a detailed calculation which demonstrates that the bouncing solution for the closed universe can appear at the minimum of scale factor. To obtain the scale factor size at the critical bouncing point, we need to take into account the behavior of the total energy density. When our universe evolves to around the critical point in high energy limit in the contracting phase, we are interested to consider the behavior of total energy density with the constant state parameter ω that takes the values in the range $(-\frac{1}{3}, \infty)$. Since in this case the cosmological constant density is very small and can be totally ignored, the behavior of the universe is governed by the dominated-content with the constant state parameter ω. Thus for any given the state parameter value ω in the allowed region, we can identify the dominated-content density with the total energy density. From the continuity equation, we can get the following density relation

$$\rho_t = \frac{C}{a^{3(1+\omega)}}, \tag{28}$$

where C is the integral constant. From the Friedmann equation(17), it gives rise to

$$(\frac{da}{dt})^2 = \frac{8\pi G}{3}a^2\rho_t(1 - \frac{\rho_t}{\rho_c}) - 1 \geq 0. \tag{29}$$

When the above equation is equal to 0, it means the Hubble parameter H is zero which is just the first bouncing condition. Substituting Eq.(28) into the above equation, we can obtain

$$36a^{4+6\omega} - 12l_p^2Ca^{3+3\omega} + \eta^2l_p^6C^2 \leq 0, \tag{30}$$

for $\rho_t < \rho_c$ and $a \geq \eta l_p$ this equation should be valid. Now let us determine the integral constant C. When putting $a = \eta l_p$ into Eq.(30), we can easily get

$$\eta^2 l_p^6 (C - 6\eta^{1+3\omega} l_p^{3\omega-1})^2 \leq 0, \tag{31}$$

on the other hand, the above equation itself has

$$\eta^2 l_p^6 (C - 6\eta^{1+3\omega} l_p^{3\omega-1})^2 \geq 0, \tag{32}$$

it is worth noting that this is the constraint exactly what we are looking for. This relation plays an important role in deciding the critical point. As a consequence, combining Eqs.(31)and(32), we have

$$C = 6\eta^{1+3\omega} l_p^{3\omega-1}. \tag{33}$$

Plugging it into Eq.(30), we find

$$a^{4+6\omega} - 2(\eta l_p)^{1+3\omega} a^{3+3\omega} + (\eta l_p)^{4+6\omega} \leq 0. \tag{34}$$

Eqs.(31)and(32) suggest that $a_0 = \eta l_p$ is a root of the Friedmann equation (17) satisfying the first bouncing condition. It is easy to check that this root saturates totally the other bouncing condition, as showed in Eq.(27). However this story is not yet end so far, since Eq.(34)generally has other roots. Therefore, in the next step, we have to show that other roots can not satisfy the second bouncing condition, otherwise the bouncing behavior occurs at the other roots. Here we provide a general trick to discuss this problem without specific state parameter ω. For convenience we define a function

$$f(a) = a^{4+6\omega} - 2(\eta l_p)^{1+3\omega} a^{3+3\omega} + (\eta l_p)^{4+6\omega}. \tag{35}$$

In order to obtain the monotonicity of this function, we need to work out the derivative of the function with respective to the scale factor a, which is

$$f' \equiv \frac{df(a)}{da} = (4+6\omega)a^{2+3\omega}[a^{1+3\omega} - \frac{3+3\omega}{2+3\omega}(\eta l_p)^{1+3\omega}] = 0, \tag{36}$$

for $\omega > -\frac{1}{3}$ and $a > 0$, we can obtain

$$\bar{a} = (\frac{3+3\omega}{2+3\omega})^{\frac{1}{1+3\omega}} \eta l_p. \tag{37}$$

Thus, we can easily check that $f(a)$ decreases monotonously, when the scale factor a in the range $[\eta l_p, \bar{a}]$; $f(a)$ increases monotonously, when the scale factor a is outside $\bar{a}$. Since

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$a_0 = \eta l_p$ is a root of $f(a) = 0$ and $f(\bar{a}) < 0$, if it has other roots, then they must be greater than $\bar{a}$.

Now we turn to considering the role of the second bouncing condition. In order to seek for the bouncing solution in normal energy region, we should consider the minus cases in both Eqs. (22)and(23). Substituting $\rho_{t-} = \frac{1}{2}\rho_c(1 - \sqrt{1 - \frac{\eta^2 l_p^2}{a_0^2}})$ into Eq.(23), we find

$$\frac{\ddot{a}_0}{a_0} = \frac{3(1 + w)}{\eta^2 l_p^2} - \frac{2 + 3w}{a_0^2} - \frac{3(1 + w)}{\eta^2 l_p^2}\sqrt{1 - \frac{\eta^2 l_p^2}{a_0^2}} \qquad (38)$$

Requiring that the above equation is greater than zero, we can derive an important constraint

$$a_0^2 < \frac{(3w + 2)^2 \eta^2 l_p^2}{(3w + 3)(3w + 1)}. \qquad (39)$$

It tells us that if any one root a_0 meets the above requirement, then the bouncing behavior can appear at the root. It is easy to check that $a_0 = \eta l_p$ satisfies this constraint. In following we will prove that the other roots do not satisfy the requirement. For the purpose, we can define a function which has a following form

$$\begin{aligned} g(w) &= \frac{\frac{(3w+2)^2 \eta^2 l_p^2}{(3w+3)(3w+1)}}{\bar{a}^2}. \\ &= \frac{(3w + 2)^{\frac{4+6w}{3w+1}}}{(3w + 3)^{\frac{3+3w}{3w+1}}(3w + 1)}, \end{aligned} \qquad (40)$$

then taking it logarithm, we have

$$\begin{aligned} y(w) &= (3w + 1)\ln g(w) \\ &= (3w + 3)[\ln(3w + 2) - \ln(3w + 3)] + (3w + 1)[\ln(3w + 2) - \ln(3w + 1)]. \end{aligned} \qquad (41)$$

To determine the monotonous behavior of this function, we can find its derivative with respective to the state parameter w.

$$\frac{dy(w)}{dw} = 3\ln\frac{(3w + 2)^2}{(3 + 3w)(1 + 3w)} > 0. \qquad (42)$$

Thus the function $y(w)$ increases monotonously in the range $(-\frac{1}{3}, \infty)$. Now we work out the behavior of the function at the infinity. Making Eq.(41)take limit, we can have

$$\begin{aligned} \lim_{w \to \infty} y(w) &= \lim_{w \to \infty}[(3w + 3)(\ln\frac{3w + 2}{3w + 3}) + (3w + 1)(\ln\frac{3w + 2}{3w + 1})] \\ &= \lim_{w \to \infty}[-\frac{1}{6 + 6w} - \frac{1}{2 + 6w} + \ldots] \\ &= 0. \end{aligned} \qquad (43)$$

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So, the function $y(\omega)$ is less than zero in $(-\frac{1}{3}, \infty)$, which means that the function $g(\omega)$ is less than 1. This tells us that although the other roots can exist, they can not saturate the second bouncing condition, namely $\ddot{a}_0 > 0$. Thus, the bouncing solution appears at $a_0 = \eta l_p$.

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